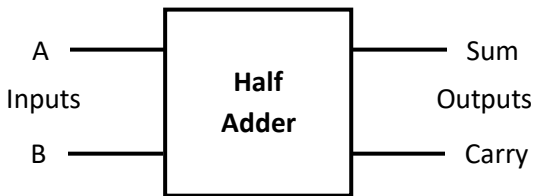


ARITHMETIC CIRCUITS

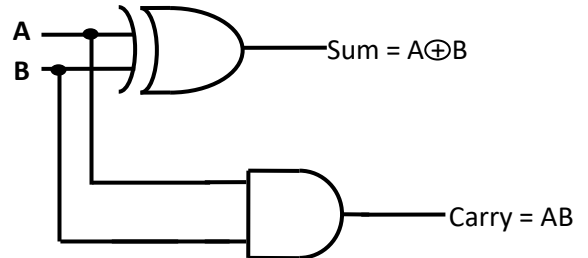
Arithmetic logic circuits are the logic circuits which perform arithmetic operations like addition, subtraction in digital computers.

Half adder: It is a combinational logic circuit which performs the addition of two bits resulting in two outputs - Sum and Carry.

Block diagram of half adder



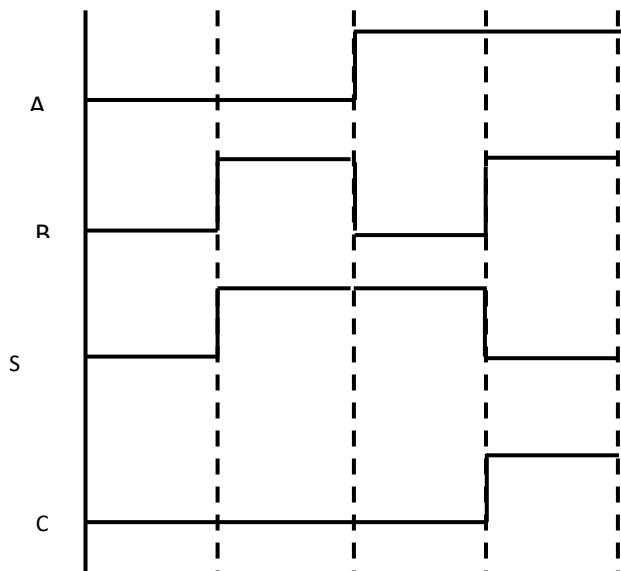
Logic diagram or logic circuit of half adder



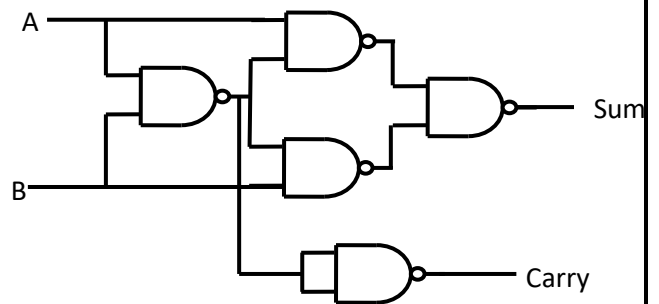
Truth table of half adder

Inputs		Outputs	
A	B	Sum = $A \oplus B$	Carry = AB
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Timing Diagram



Half adder using universal NAND gates



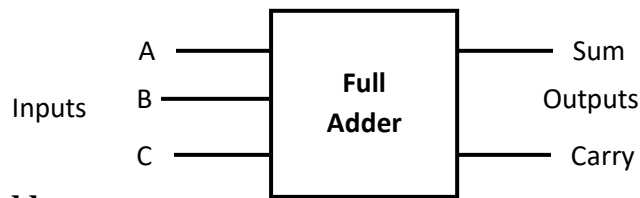
Boolean expression:

$$\text{Sum} = A \oplus B = \bar{A} B + A \bar{B}$$

$$\text{Carry} = AB$$

Full adder: It is a combinational logic circuit which performs the addition of three bits resulting in two outputs - Sum and Carry.

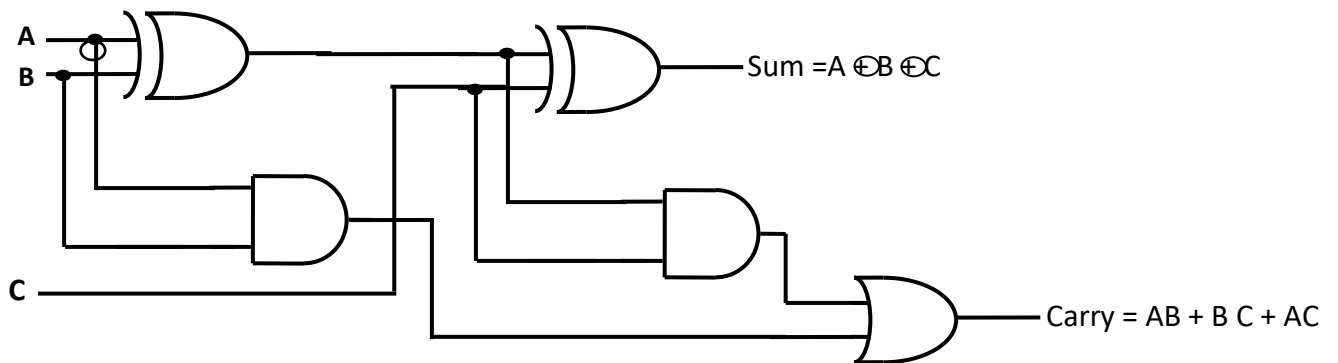
Block diagram of full adder



Truth table of full adder

Inputs			Outputs	
A	B	C	Sum = $A \oplus B \oplus C$	Carry = $AB + BC + CA$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Logic circuit of full adder



Boolean expression for the sum

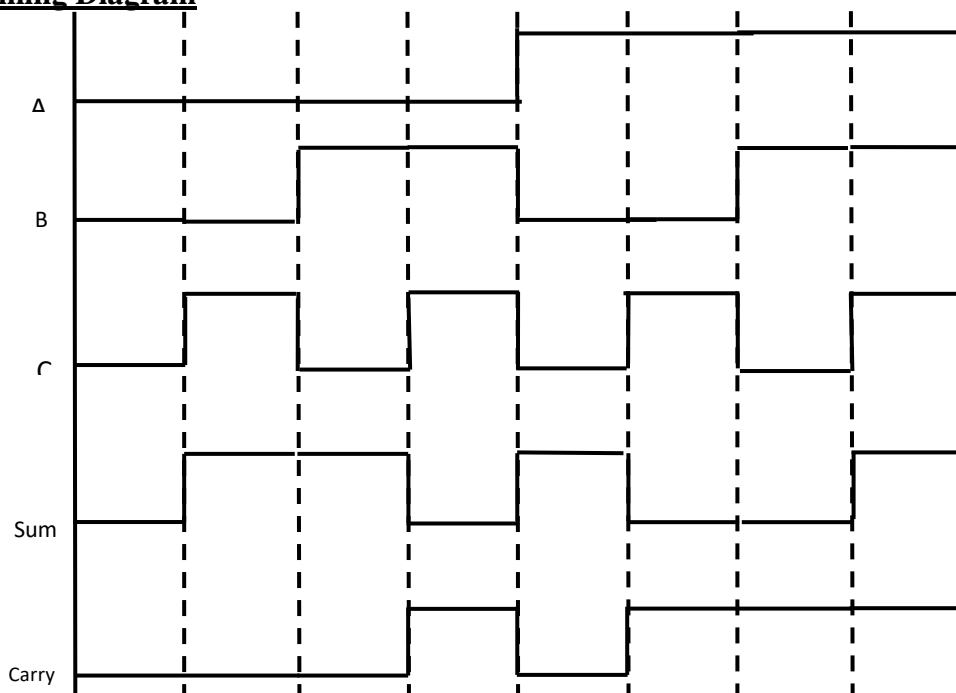
$$\text{Sum} = A \oplus B \oplus C$$

$$\text{Sum} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

Boolean expression for the carry

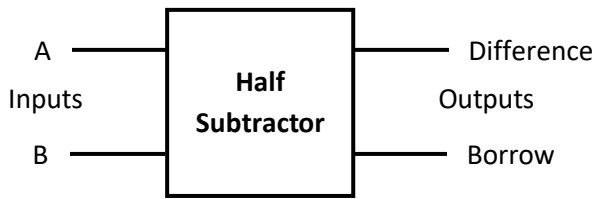
$$\text{Carry} = AB + BC + AC$$

Timing Diagram

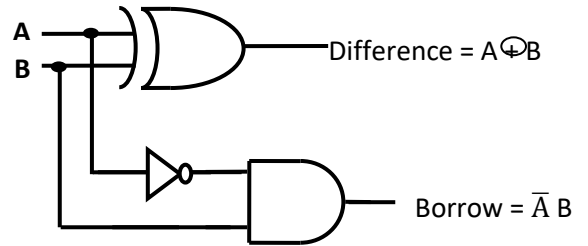


Half Subtractor: It is combinational logic circuits which performs the subtraction of two bits resulting in two outputs - Difference and Borrow.

Block diagram of half Subtractor



Logic diagram or logic circuit of half ubtractor



Truth table of half Subtractor

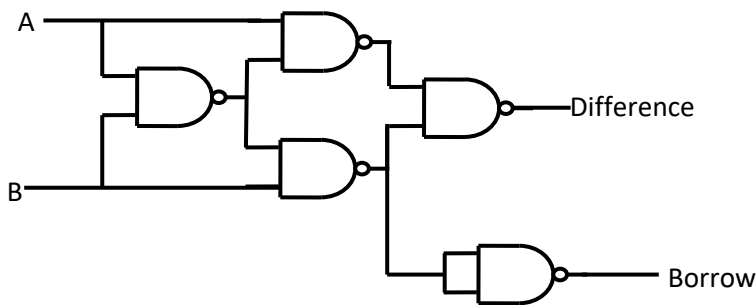
Inputs		Outputs	
A	B	Difference = $A \oplus B$	Borrow = $\bar{A} B$
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Boolean expression:

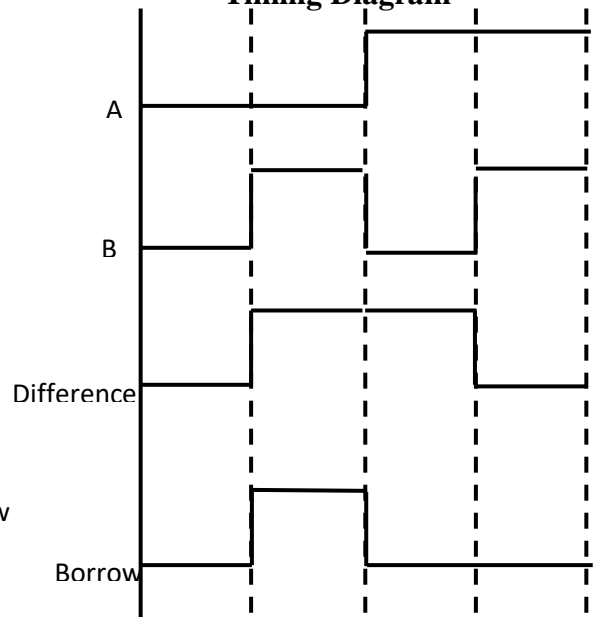
Difference = $A \oplus B = \bar{A} B + A \bar{B}$

Borrow = $\bar{A} B$

Half subtractor using universal NAND gates

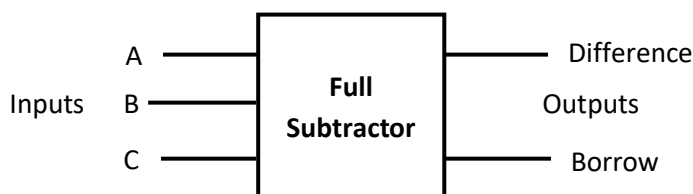


Timing Diagram



Full subtractor: It is combinational logic circuits which performs the subtraction of three bits resulting in two outputs difference and borrow.

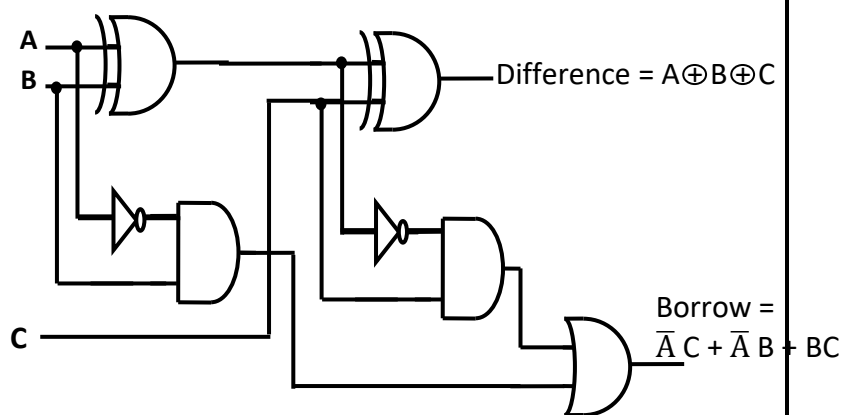
Block diagram of full Subtractor



Truth table of full subtractor

Input			Output	
A	B	C	Difference $= A \oplus B \oplus C$	Borrow $= \bar{A}C + \bar{A}B + BC$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

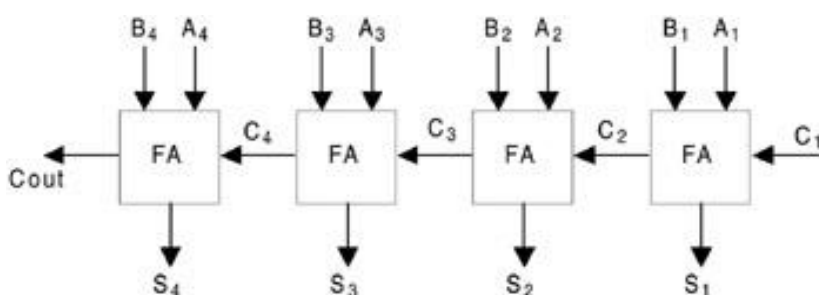
Logic circuit of full subtractor



4 BIT PARALLEL BINARY ADDER:

A **Parallel Adder** is a digital circuit capable of finding the arithmetic **sum** of two binary numbers that is **greater than one bit** in length by operating on corresponding pairs of bits in parallel. It consists of **full adders connected in cascade** where the output carry from each full adder is connected to the carry input of the next higher order full adder

Consider two 4-bit binary numbers $B_4 B_3 B_2 B_1$ and $A_4 A_3 A_2 A_1$ are to be added with a carry input C_1 . This can be done by cascading four full adder circuits as shown in Figure. The least significant bits A_1, B_1 , and C_1 are added to produce sum output S_1 and carry output C_2 . Carry output C_2 is then added to the next significant bits A_2 and B_2 producing sum output S_2 and carry output C_3 . C_3 is then added to A_3 and B_3 and so on. Thus finally producing the four-bit sum output $S_4 S_3 S_2 S_1$ and final carry output C_{out} .

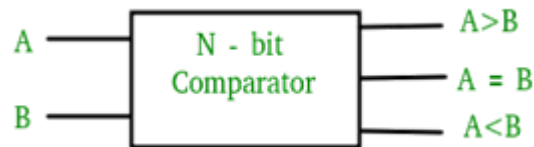


Magnitude Comparator

Data comparison is needed in digital systems while performing arithmetic or logical operations. This comparison determines whether one number is greater than, equal, or less than the other number.

A Magnitude Comparator is a combinational circuit that **compares two binary numbers** in order to find out whether one binary number is equal, less than or greater than the other binary number.

The Block diagram of Magnitude Comparator is as shown below.



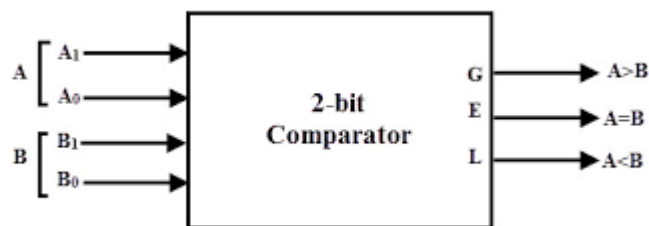
Types of Magnitude Comparator:

1. 1-bit magnitude comparator.
2. 2-bit magnitude comparator.
3. 3-bit magnitude comparator.
4. 4-bit magnitude comparator.

2-bit magnitude comparator:

A 2-bit comparator compares two binary numbers, each of two bits and produces their relation such as one number is equal or greater than or less than the other. The figure below shows the block diagram of a two-bit comparator which has four inputs and three outputs.

The first number A is designated as $A = A_1A_0$ and the second number is designated as $B = B_1B_0$. This comparator produces three outputs as G ($G = 1$ if $A > B$), E ($E = 1$, if $A = B$) and L ($L = 1$ if $A < B$).



The truth table for a 2-bit comparator is given below:

Inputs				Outputs		
A ₁	A ₀	B ₁	B ₀	A>B	A=B	A<B
0	0	0	0	0	1	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	1	0	0
0	1	0	1	0	1	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	0	1	1	0	0	1
1	1	0	0	1	0	0
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	0	1	0

4-Bit Comparator

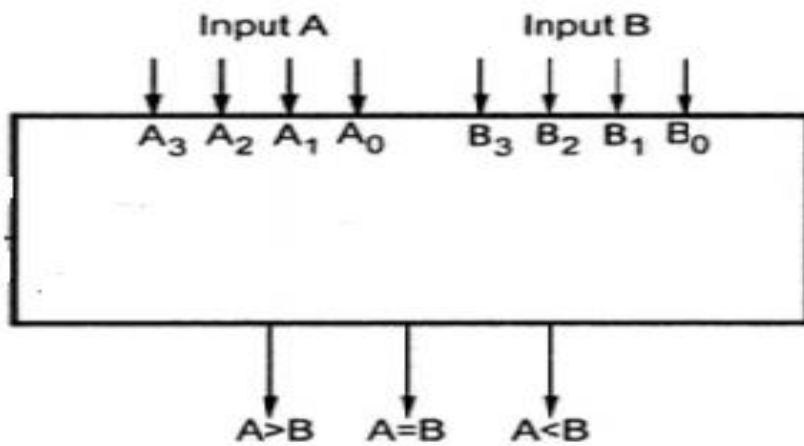
It can be used to compare two four-bit words. The two 4-bit numbers are $A = A_3 A_2 A_1 A_0$ and $B_3 B_2 B_1 B_0$ where A_3 and B_3 are the most significant bits.

It compares each of these bits in one number with bits in that of other number and produces one of the following outputs as $A = B$, $A < B$ and $A > B$.

The output logic statements of this converter are

- If $A_3 = 1$ and $B_3 = 0$, then A is greater than B ($A > B$). Or
- If A_3 and B_3 are equal, and if $A_2 = 1$ and $B_2 = 0$, then $A > B$. Or
- If A_3 and B_3 are equal & A_2 and B_2 are equal, and if $A_1 = 1$, and $B_1 = 0$, then $A > B$. Or
- If A_3 and B_3 are equal, A_2 and B_2 are equal and A_1 and B_1 are equal, and if $A_0 = 1$ and $B_0 = 0$, then $A > B$.

Block diagram:



Truth table

COMPARING INPUTS				OUTPUT		
A ₃ , B ₃	A ₂ , B ₂	A ₁ , B ₁	A ₀ , B ₀	A > B	A < B	A = B
A ₃ > B ₃	X	X	X	H	L	L
A ₃ < B ₃	X	X	X	L	H	L
A ₃ = B ₃	A ₂ > B ₂	X	X	H	L	L
A ₃ = B ₃	A ₂ < B ₂	X	X	L	H	L
A ₃ = B ₃	A ₂ = B ₂	A ₁ > B ₁	X	H	L	L
A ₃ = B ₃	A ₂ = B ₂	A ₁ < B ₁	X	L	H	L
A ₃ = B ₃	A ₂ = B ₂	A ₁ = B ₁	A ₀ > B ₀	H	L	L
A ₃ = B ₃	A ₂ = B ₂	A ₁ = B ₁	A ₀ < B ₀	L	H	L
A ₃ = B ₃	A ₂ = B ₂	A ₁ = B ₁	A ₀ = B ₀	H	L	L
A ₃ = B ₃	A ₂ = B ₂	A ₁ = B ₁	A ₀ = B ₀	L	H	L
A ₃ = B ₃	A ₂ = B ₂	A ₁ = B ₁	A ₀ = B ₀	L	L	H

H = High Voltage Level, L = Low Voltage, Level, X = Don't Care

Reference Books:

1. Thomas L. Floyd, "Digital Fundamentals," 11th Edition, Pearson Education, 2015.
2. A.P. Malvino, D. P. Leach, and Saha, "Digital Principles and Applications," 8th Edition, TMH, 2014.
3. https://www.electronics-tutorials.ws/combination/comb_8.html
4. <https://www.elprocus.com/digital-comparator-and-magnitude-comparator/>